KEK-TH-1152
OIQP-07-07
YITP-07-29

Fluxes of Higher-spin Currents and Hawking Radiations from Charged Black Holes

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Abstract

This is an extended version of the previous paper (hep-th/0701272). Quantum fields near horizons can be described in terms of an infinite set of two-dimensional conformal fields. We first generalize the method of Christensen and Fulling to charged black holes to derive fluxes of energy and charge. These fluxes can be obtained by employing a conformal field theory technique. We then apply this technique to obtain the fluxes of higher-spin currents and show that the thermal distribution of Hawking radiation from a charged black hole can be completely reproduced by investigating transformation properties of the higher-spin currents under conformal and gauge transformations.

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1 Introduction

Hawking radiation is a characteristic quantum effect which arises in the background space-time with event horizons [1,2]. The radiation has the spectrum with the Planck distribution and it gives one of the thermodynamic properties of black holes. Hawking studied quantum systems of matter in collapsing geometries and derived the thermal radiation. Soon after Unruh revealed importance of choice of vacua, i.e. choice of boundary conditions, and showed that eternal black holes also emit the same radiation [3]. Since then various derivations were proposed, all of which lead to the same result.

Recently Robinson and Wilczek proposed a new derivation of Hawking radiation [4]. They realized that effective theories of matter fields become two-dimensional and chiral near the event horizon of a Schwarzschild black hole because of the structures proper to the horizon. Then they showed that the existence of energy flux of the Hawking radiation is required for the cancellation of the gravitational anomalies near the horizon. The method was then generalized to charged black holes [5] by using the gauge anomaly in addition to the gravitational anomaly and further to rotating black holes [6, 7] and other various cases [8–20]. In these analyses, it is essential that each partial wave mode of quantum fields near the horizon behaves as a two-dimensional field [21]. Then, near the horizon, outgoing modes are interpreted as right moving modes and ingoing modes as left moving modes. Since the ingoing modes immediately fall into the interior region of the black hole and never come out, they are considered to be classically irrelevant to physics outside the black hole. However these modes can not be neglected at the quantum level because if the ingoing modes do not exist the theory becomes chiral and thus anomalous under the diffeomorphism and gauge transformations. The fluxes of energy and charge from the Hawking radiation are determined by vanishing conditions of the anomalies at the horizon.

The method based on the gravitational and gauge anomalies successfully reproduce the Hawking fluxes as mentioned above, but the fluxes of energy and charge are merely a part of the information of the thermal distribution. In Ref [22] we proposed a method using conformal field theory technique to reconstruct the full thermal spectrum in a Schwarzschild black hole. We there used the fact that each moment of the Planck distribution, which is an integration of the distribution function multiplied by an appropriate power of frequency, is equivalent to a flux of the corresponding higher-spin current. The fluxes are given by expectation values of the higher-spin currents which are defined with respect to the coordinate system appropriate at infinity. These quantities at infinity are related to the ones in the Kruskal coordinates by a conformal transformation. We evaluated anomalous terms in transformation properties of the higher-spin currents under the conformal transformation. These terms give main contributions to the fluxes. The fluxes are deter-

mined by imposing the regularity condition that the expectation values of the currents should be regular at the horizon in the Kruskal coordinates.

In this paper we generalize the method to charged black holes by considering a gauge transformation in addition to the conformal transformation. In a gauge that the background $U(1)$ gauge field vanishes at infinity, the gauge potential is singular at the horizon. In order to adequately impose the regularity condition on quantities at the horizon, we have to use a gauge in which the background field behaves regularly there. Hence we need to know the transformation properties of the higher-spin currents under suitable gauge and conformal transformations, which map physical quantities near the horizon to those at infinity. We will derive the fluxes of the higher-spin currents containing the complete information of the thermal spectrum of the Hawking radiation by using such relations and the regularity condition.

This paper is organized as follows. In section 2 we make a generalization of the method by Christensen and Fulling [23] to charged black holes. Fluxes of energy and charge are obtained by solving conservation equations of energy-momentum tensor and $U(1)$ current and the anomaly equations for conformal and chiral symmetries. In section 3, we derive the same fluxes by using conformal field theory technique. This analysis is applied to higher-spin currents to derive their fluxes. We conclude in section 5.

2 Christensen-Fulling method for charged black holes

Christensen and Fulling obtained the flux of Hawking radiation from neutral black holes by solving the conservation equations of the energy-momentum tensor together with the information of the trace anomaly [23]. In their seminal paper, they showed that the outgoing flux at infinity cannot be put zero if we require (1) conservation of energy-momentum tensor (2) trace anomaly and (3) regularity at the future horizon. Their calculation works well in the two-dimensional case but in higher dimensions, since the number of components of the energy-momentum tensor is larger than the above three information, all the components cannot be determined fully without further information. Also a generalization to charged (or rotating) black holes needs further information to determine the flux. In this section, we generalize the method by Christensen and Fulling to two-dimensional charged black holes by further considering the conservation of the gauge current and the chiral anomaly equation.

2.1 Reissner-Nordström black hole

We first summarize the basics of Reissner-Nordström black holes. The metric and the gauge potential of Reissner-Nordström black holes with mass M and charge Q are given by

$$\begin{aligned} ds^2 &= f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2 d\Omega_2^2, && (2.1) \\ A_t &= -\frac{Q}{r}, && (2.2) \end{aligned}$$

where

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = \frac{(r-r_+)(r-r_-)}{r^2} \tag{2.3}$$

and the radius of outer (inner) horizon $r_\pm$ is given by

$$r_\pm = M \pm \sqrt{M^2 - Q^2}. \tag{2.4}$$

It is useful to define the tortoise coordinate by solving $dr_* = dr/f$ as

$$r_* = r + \frac{1}{2\kappa_+} \ln \frac{|r - r_+|}{r_+} + \frac{1}{2\kappa_-} \ln \frac{|r - r_-|}{r_-}. \tag{2.5}$$

Here the surface gravity at $r_\pm$ is given by

$$\kappa_\pm = \frac{1}{2} f'(r_\pm) = \frac{r_\pm - r_\mp}{2r_\pm^2}. \tag{2.6}$$

In the following we consider near the outer horizon. First we define the light-cone coordinates, $u = t - r_*$ and $v = t + r_*$. $u(v)$ are the outgoing (ingoing) coordinates and the metric in these coordinates becomes as

$$ds^2 = f(dt^2 - dr_*^2) - r^2 d\Omega^2 = f du dv - r^2 d\Omega^2. \tag{2.7}$$

In order to investigate the physics near the outer horizon, since (u, v) coordinate is still singular at the horizon, it is important to introduce a regular coordinate, Kruskal coordinate, defined by the transformations

$$U = -e^{-\kappa_+ u}, \qquad V = e^{\kappa_+ v}. \tag{2.8}$$

The metric becomes

$$ds^2 = \frac{r_+ r_-}{\kappa_+^2} \frac{1}{r^2} e^{-2\kappa_+ r} \left(\frac{r_-}{r - r_-} \right)^{\frac{\kappa_+}{\kappa_-} - 1} dU dV - r^2 d\Omega^2. \tag{2.9}$$

If we restrict to see the two-dimensional (r,t) section, both of these coordinates (2.7), (2.9) have the forms of the conformal gauge

$$ds^2 = e^{\varphi(u,v)}dudv = e^{\varphi'(U,V)}dUdV. \tag{2.10}$$

The transformation (2.8) is a conformal transformation from an asymptotically flat coordinate to a regular coordinate near the horizon.

Hereafter we omit the subscript $+$ of the surface gravity κ_+ at the outer horizon because κ_- does not appear in the following analysis.

2.2 Solving conservation equations

We now solve conservation equations for the gauge current and the energy-momentum tensor together with the information of trace and chiral anomalies. Here we consider a charged matter field in two-dimensional black holes. If we get a two-dimensional system by a dimensional reduction of higher-dimensional ones, we need to take care of the effects of angular components of the gauge current or the energy-momentum tensor. In the following, we simply neglect these effects and consider purely two-dimensional cases for simplicity. Then conservation laws for the gauge current and the energy-momentum tensor in the background of gravitational and gauge fields are given by

$$\nabla_\mu T^\mu_\nu = F_{\mu\nu}J^\mu, \tag{2.11}$$
$$\nabla_\mu J^\mu = 0. \tag{2.12}$$

If we consider conformal matter fields in this background, the energy-momentum tensor has the trace anomaly

$$T^\mu_\mu = \frac{c}{24\pi}R, \tag{2.13}$$

where c is the central charge of the conformal fields, $R = -4e^{-\varphi}\partial_u\partial_v\varphi$ is a scalar curvature and the conformal factor φ in the (u,v) coordinate is given by $\varphi = \ln f(r)$. The two-dimensional chiral current, which is related to the gauge current by $J^{5\mu} = \epsilon^{\mu\nu}J_\nu$, has an anomaly. Here $\epsilon^{\mu\nu}$ is a covariant antisymmetric tensor, $\epsilon^{uv} = 2e^{-\varphi}$. For a fermion field with charge e, it is given by

$$\nabla_\mu J^{5\mu} = \frac{e^2}{2\pi}\epsilon^{\mu\nu}F_{\mu\nu}. \tag{2.14}$$

For more general cases, the coefficients depend on the matter contents but in the following we absorb them into the charge e and use the same conservation equation (2.14).

First let us solve the conservation equations for gauge (2.12) and chiral currents (2.14). By using the relation $J^{5\mu} = \epsilon^{\mu\nu}J_\nu$, these two equations can be solved as

$$J_u = j(u) + \frac{e^2}{\pi}A_u, \qquad J_v = \tilde{j}(v) + \frac{e^2}{\pi}A_v. \tag{2.15}$$

where $j(u)$ (or $\tilde{j}(v)$) is a holomorphic (or anti-holomorphic) function satisfying $\partial_v j(u) = \partial_u \tilde{j}(v) = 0$. As we see in the next section, these (anti-) holomorphic currents play important roles in the conformal field theories. Note that they are different from the ordinary currents J_u, J_v by a term proportional to the background gauge potential and not invariant under gauge transformations.

In order to determine these functions in the black hole background, we use the same boundary conditions as those used in the paper by Christensen and Fulling. Namely, for an outgoing flux $j(u)$, we impose that the physics should be regular in the Kruskal coordinate U. Since $J_U = J_u/(-\kappa U)$, the flux J_u must vanish at the future outer horizon $U = 0$. Thus $j(u)$ is determined to be

$$j(u) = -\frac{e^2}{\pi} A_u(r_+). \tag{2.16}$$

The anti-holomorphic part $\tilde{j}(v)$ is determined by the condition that there is no ingoing flux from infinity ($r \to \infty$) and given by

$$\tilde{j}(v) = 0. \tag{2.17}$$

These two conditions correspond to taking the so-called Unruh vacuum. Hence the radial component of the electromagnetic current in the charged black hole is given by

$$J^r = J_u - J_v = -\frac{e^2}{\pi} A_u(r_+) = \frac{e^2 Q}{2\pi r_+}. \tag{2.18}$$

Similarly the expectation value of the energy-momentum tensor in the charged black hole can be determined by solving eqs.(2.11) and (2.13). The uu and vv components of them can be solved as

$$\begin{aligned} T_{uu} &= t(u) + \frac{c}{24\pi}\left(\partial_u^2\varphi - \frac{1}{2}(\partial_u\varphi)^2\right) + \frac{e^2}{\pi}A_u^2 + 2A_u j(u), & (2.19) \\ T_{vv} &= \tilde{t}(v) + \frac{c}{24\pi}\left(\partial_v^2\varphi - \frac{1}{2}(\partial_v\varphi)^2\right) + \frac{e^2}{\pi}A_v^2 + 2A_v \tilde{j}(v). & (2.20) \end{aligned}$$

Here $t(u)$ is a holomorphic function and $\tilde{t}(v)$ is an anti-holomorphic function, which can be determined by the following boundary conditions.

Imposing the regularity condition at the outer horizon, T_{uu} must vanish at the future outer horizon $U = 0$. Hence $t(u)$ is determined as

$$t(u) = \frac{c}{192\pi}\left(f'(r_+)\right)^2 + \frac{e^2}{\pi} A_u^2(r_+). \tag{2.21}$$

Similarly $\tilde{t}(v)$ is determined by requiring that there is no ingoing flux from infinity as

$$\tilde{t}(v) = 0. \tag{2.22}$$

Hence the rt-component of the energy-momentum tensor T_t^r is obtained as

$$T_t^r = T_{uu} - T_{vv} = -\frac{e^2}{2\pi} A_t(r_+)A_t(r) + \frac{c}{192\pi}\left(f'(r_+)\right)^2 + \frac{e^2}{4\pi} A_t^2(r_+). \tag{2.23}$$

The asymptotic flux of the energy is given by the asymptotic value of T_t^r at $r \to \infty$,

$$T_t^r \longrightarrow \frac{c}{192\pi}\left(f'(r_+)\right)^2 + \frac{e^2}{4\pi} A_t^2(r_+). \tag{2.24}$$

The flux reproduces the correct flux of Hawking radiation from the charged black hole [5].

3 U(1) current and energy-momentum tensor

In section 2, we generalized the method by Christensen and Fulling to charged black holes and obtained the fluxes of charge and energy. In this section we derive the same result by using a technique of conformal field theory without resorting to the covariant calculation done in section 2. Here we consider a charged fermionic field in the black hole background. It is also easy to apply the method to a bosonic field.

We define the holomorphic $U(1)$ current $:\psi^\dagger(u)\psi(u):$ by using the point splitting regularization as follows [24],

$$:\psi^\dagger(u)\psi(u): \equiv \lim_{\epsilon\to 0}\left(\psi^\dagger(u+\epsilon)\psi(u) + \frac{i}{2\pi\epsilon}\right). \tag{3.1}$$

Here we used $\psi(z)\psi^\dagger(w) \sim -i/(2\pi(z-w))$. Under a holomorphic $U(1)$ transformation*, the fermion transforms as $\psi^{(\Lambda)}(u) = e^{ie\Lambda(u)}\psi(u)$. Then the transformation law for the current can be calculated as

$$\begin{aligned} :\psi^{(\Lambda)\dagger}(u)\psi^{(\Lambda)}(u): &= \lim_{\epsilon\to 0}\left(e^{-ie\Lambda(u+\epsilon)+ie\Lambda(u)}\psi^\dagger(u+\epsilon)\psi(u) + \frac{i}{2\pi\epsilon}\right) \\ &= \lim_{\epsilon\to 0}\left[e^{-ie\Lambda(u+\epsilon)+ie\Lambda(u)}\left(:\psi^\dagger(u+\epsilon)\psi(u): - \frac{i}{2\pi\epsilon}\right) + \frac{i}{2\pi\epsilon}\right] \\ &= \ :\psi^\dagger(u)\psi(u): - \frac{e}{2\pi}\partial_u\Lambda(u). \end{aligned} \tag{3.2}$$

Note that, since we have defined the current by the point splitting method without introducing a Wilson line, the current is not invariant under the gauge transformation. The current is identified with the holomorphic current $j(u)$ defined in (2.15), which is also not gauge invariant, as $j(u) = e :\psi^\dagger(u)\psi(u):$.

Under a conformal transformation $u \to w(u)$, the fermion $\psi(u)$ transforms as $\psi(u) = (\partial_u w(u))^{1/2}\psi^{(w)}(w(u))$. Hence the current transforms as a tensor with a conformal weight 1 as expected;

$$:\psi^\dagger(u)\psi(u): = \partial_u w(u) :\psi^{(w)\dagger}(w(u))\psi^{(w)}(w(u)):. \tag{3.3}$$

* A holomorphic transformation δ_h with an infinitesimal parameter $\lambda(u)$ is a combination of a gauge transformation δ_G and a chiral transformation δ_C with the same parameter $\lambda(u)$; $\delta_h = \frac{1}{2}(\delta_G - \delta_C)$.

We apply these transformation properties (3.2) and (3.3) to the charged black hole background. Unruh vacuum is defined by using modes associated with the Kruskal coordinate $U = -e^{-\kappa u}$, which is regular at the outer horizon. (i.e. Unruh vacuum is annihilated by positive frequency modes defined in terms of the Kruskal coordinate.) Furthermore the background gauge potential (2.2) is singular at the outer horizon in the Kruskal coordinates because $A_U = (-1/\kappa U)A_u = (-1/2\kappa U)A_t$, $(A_r = 0)$. In order to impose boundary conditions for currents, we need to take a gauge which is regular at the outer horizon $U = 0$ in the Kruskal coordinates. Such a gauge choice is given by $A_t = -Q/r + Q/r_+$. We denote the gauge fields in two different gauges as

$$A_t^{(u)} = -\frac{Q}{r}, \qquad A_t^{(U)} = -\frac{Q}{r} + \frac{Q}{r_+}. \tag{3.4}$$

The first gauge potential vanishes at $r \to \infty$ and it is appropriate to use it to measure the frequency at an asymptotic infinity. Hence we use the superscript (u). The second one does not satisfy this property but it vanishes at the outer horizon and the corresponding A_U behaves regularly there. This is why we use the superscript (U) for this gauge. They are related by a gauge transformation with a parameter tQ/r_+; $A_t^{(U)} = A_t^{(u)} + \partial_t(tQ/r_+)$. In the holomorphic sector, we consider a holomorphic $U(1)$ transformation with a parameter $\Lambda(u) = uQ/r_+$ to implement this gauge transformation†. Hereafter we explicitly indicate the gauge dependence of expectation values of (gauge-dependent) operators by using the subscripts $A^{(u)}$ or $A^{(U)}$ as $\langle \mathcal{O} \rangle_{A^{(u)}}$ and $\langle \mathcal{O} \rangle_{A^{(U)}}$.

First the expectation values of the current in these gauges, $\langle : \psi^\dagger(u)\psi(u) : \rangle_{A^{(u)}}$ and $\langle : {\psi^{(U)}}^\dagger(U)\psi^{(U)}(U) : \rangle_{A^{(U)}}$, are related as follows;

$$\begin{aligned} \langle : \psi^\dagger(u)\psi(u) : \rangle_{A^{(u)}} &= \langle : \psi^\dagger(u)\psi(u) : \rangle_{A^{(U)}} + \frac{e}{2\pi}\partial_u \Lambda(u) \\ &= -\kappa U \langle : {\psi^{(U)}}^\dagger(U)\psi^{(U)}(U) : \rangle_{A^{(U)}} + \frac{e}{2\pi}\partial_u \Lambda(u). \end{aligned} \tag{3.5}$$

In the second line we make a conformal transformation from u coordinate to U. We impose the boundary condition that physical quantities should behave regularly at the outer future horizon $U = 0$ in the Kruskal coordinate U and the gauge $A^{(U)}$ which is regular there. In other words, $\langle : {\psi^{(U)}}^\dagger(U)\psi^{(U)}(U) : \rangle_{A^{(U)}}$ should be finite at $U = 0$. This condition determines the flux of the charge current at infinity to be

$$\langle : \psi^\dagger(u)\psi(u) : \rangle_{A^{(u)}} = \frac{eQ}{2\pi r_+}, \tag{3.6}$$

† Our choice of $\Lambda(u)$ seems to be twice as large as the one required by the gauge transformation since $\frac{tQ}{r_+} = \frac{uQ}{2r_+} + \frac{vQ}{2r_+}$. This factor of two arises from a relation between the fermionic field used to construct the conformal current and the original fermionic field in the gauge field background. We can show that by taking this field redefinition into account the relations between the covariant and conformal currents (eqs.(2.15) or (2.19)) can be correctly obtained. We will give further discussions on this issue in our future publication [25].

where we have used $\Lambda(u) = uQ/r_+$. This coincides with (2.16) obtained by solving the conservation equations.

The energy-momentum tensor is defined similarly by the following point splitting method,

$$
\begin{aligned}
&: \frac{i}{2}\left(\psi^\dagger(u)\partial_u\psi(u) - \partial_u\psi^\dagger(u)\psi(u)\right) : \\
&\qquad \equiv \lim_{\epsilon\to 0}\left[\frac{i}{2}\left(\psi^\dagger(u+\epsilon)\partial_u\psi(u) - \partial_u\psi^\dagger(u+\epsilon)\psi(u)\right) - \frac{1}{2\pi\epsilon^2}\right]. \qquad (3.7)
\end{aligned}
$$

Under a conformal transformation, the energy-momentum tensor transforms as

$$
\begin{aligned}
&: \frac{i}{2}\left(\psi^\dagger(u)\partial_u\psi(u) - \partial_u\psi^\dagger(u)\psi(u)\right) : \\
&\quad = (\partial_u w(u))^2 : \frac{i}{2}\left(\psi^{(w)\dagger}(w(u))\partial_w\psi^{(w)}(w(u)) - \partial_w\psi^{(w)\dagger}(w(u))\psi^{(w)}(w(u))\right) : \\
&\quad -\frac{1}{24\pi}\{w,u\}^{(1)}, \qquad (3.8)
\end{aligned}
$$

where $\{w,u\}^{(1)}$ is the Schwarzian derivative

$$
\{w,u\}^{(1)} = \frac{\partial_u^3 w(u)}{\partial_u w(u)} - \frac{3}{2}\left(\frac{\partial_u^2 w(u)}{\partial_u w(u)}\right)^2. \qquad (3.9)
$$

This energy-momentum tensor corresponds to the holomorphic tensor $t(u)$ defined in (2.19). The extra factor in the transformation (Schwarzian derivative) comes from the Weyl transformation of the second term in the right hand side of (2.19).

Under the $U(1)$ transformation, the energy-momentum tensor (3.7) transforms as

$$
\begin{aligned}
&: \frac{i}{2}\left(\psi^{(\Lambda)\dagger}(w(u))\partial_u\psi^{(\Lambda)}(u) - \partial_u\psi^{(\Lambda)\dagger}(u)\psi^{(\Lambda)}(u)\right) : \\
&\qquad =: \frac{i}{2}\left(\psi^\dagger(u)\partial_u\psi(u) - \partial_u\psi^\dagger(u)\psi(u)\right) : -e\partial_u\Lambda(u) : \psi^\dagger(u)\psi(u) : \\
&\qquad +\frac{e^2}{4\pi}\left(\partial_u\Lambda(u)\right)^2. \qquad (3.10)
\end{aligned}
$$

This is not gauge invariant, which can be understood either from the gauge dependent factors in (2.19) or from the point splitting definition without a Wilson line (3.7).

Now we apply these transformations (3.8) and (3.10) to derive the expectation values of the energy-momentum tensor in the charged black hole background. The energy-momentum tensor in the u coordinate with the gauge $A^{(u)}$ is related to that in the U coordinate with the gauge $A^{(U)}$ as

$$
\begin{aligned}
&\langle : \frac{i}{2}\left(\psi^\dagger(u)\partial_u\psi(u) - \partial_u\psi^\dagger(u)\psi(u)\right) : \rangle_{A^{(u)}} \\
&\quad = (\kappa U)^2 \langle : \frac{i}{2}\left(\psi^{(U)\dagger}(U)\partial_U\psi^{(U)}(U) - \partial_U\psi^{(U)\dagger}(U)\psi^{(U)}(U)\right) : \rangle_{A^{(U)}} \\
&\qquad +e\partial_u\Lambda(u)(\kappa U)\langle : \psi^{(U)\dagger}(U)\psi^{(U)}(U) : \rangle_{A^{(U)}} - \frac{1}{24\pi}\{U,u\} + \frac{e^2}{4\pi}\left(\partial_u\Lambda(u)\right)^2. \qquad (3.11)
\end{aligned}
$$

Hence, by imposing the regularity condition for T_{UU} in the $A^{(U)}$ gauge, we have

$$\langle : \frac{i}{2}\left(\psi^\dagger(u)\partial_u\psi(u) - \partial_u\psi^\dagger(u)\psi(u)\right) :\rangle_{A^{(u)}} = \frac{\kappa^2}{48\pi} + \frac{e^2Q^2}{4\pi r_+^2}. \tag{3.12}$$

This value is equal to (2.21) in the previous section.

4 Higher-spin currents

We now generalize the method in the previous section to higher-spin currents. The fluxes of these currents correspond to higher moments of the frequency integral of the Hawking radiation. Hence, if the fluxes of all types of higher-spin currents are obtained, we can fully reproduce the thermal spectrum of Hawking radiations from (charged) black holes.

In order to get the fluxes of charge and energy flows, we used a covariant approach in section 2 and a conformal field theory approach in section 3. As we saw, they are equivalent but the latter approach is much simpler since we do not need either to know the covariant formulation of the (trace or chiral) anomaly equations or to solve the conservation equations of currents. But there is an issue which should be clarified to generalize the above approach to cases of higher-spin currents. We need to know relations between covariant higher-spin currents and corresponding (anti-)holomorphic quantities like (2.15) and (2.19) in order to show that expectation values derived by using conformal field theory technique adopted here coincide with fluxes obtained by covariant calculation. In this paper, we use the fact, sufficient for the present analysis, that differences between covariant higher-spin currents and holomorphic ones approach to 0 at infinity. Then we show that fluxes derived by conformal field theory approach coincide with certain moments of the Fermi-Dirac distribution of the Hawking radiations, which will be defined below. We will give full discussions on this issue in our future publication [25].

The holomorphic component of the $(n+1)$-th rank higher-spin current is given by a linear combination of $\partial_u^m\psi^\dagger\partial_u^{n-m}\psi$. Since we can show that all these terms give the same contribution to an expectation value of the current in the black hole background [22], we here consider only $\psi^\dagger\partial_u^n\psi$. First we introduce higher-spin currents which are regularized by the point splitting as follows,

$$:\psi^\dagger(u)\partial_u^n\psi(u) :\equiv \lim_{\epsilon\to 0}\left[\psi^\dagger(u+\epsilon)\partial_u^n\psi(u) + \frac{in!}{2\pi\epsilon^{n+1}}\right]. \tag{4.1}$$

It is convenient to introduce a generating function of the higher-spin currents,

$$:\psi^\dagger(u)\psi(u+a) :\equiv \sum_{n=0}^{\infty}\frac{a^n}{n!} :\psi^\dagger(u)\partial_u^n\psi(u) : . \tag{4.2}$$

This generating function is defined by the right hand side and should be understood as a formal power series of a whose coefficients are normal ordered operators located at u. The fermion has a conformal weight 1/2 and ψ transforms as $\psi(u) = (\partial_u w(u))^{1/2}\psi^{(w)}(w(u))$ under a conformal transformation $u \to w(u)$. Therefore the generating function transforms as

$$\begin{aligned}
:\psi^\dagger(u)\psi(u+a): &= \psi^\dagger(u)\psi(u+a) - \frac{i}{2\pi a} \\
&= [\partial_u w(u)\partial_u w(u+a)]^{1/2} :\psi^{(w)\dagger}(w(u))\psi^{(w)}(w(u+a)): \\
&\quad +A_f(w,u).
\end{aligned} \tag{4.3}$$

Here $A_f(w,u)$ is a generating function of generalized Schwarzian derivatives for higher-spin currents (4.2) and given by

$$A_f(w,u) = \sum_{n=0}^{\infty} \frac{(-ia)^n}{n!}\{w,u\}^{(n)} = \frac{i}{2\pi}\left(\frac{[\partial_u w(u)\partial_u w(u+a)]^{1/2}}{w(u+a)-w(u)} - \frac{1}{a}\right). \tag{4.4}$$

$\{w,u\}^{(1)}$ is proportional to the ordinary Schwarzian derivative.

Next under the $U(1)$ transformation $\psi^{(\Lambda)}(u) = e^{ie\Lambda(u)}\psi(u)$, the generating function transforms as

$$\begin{aligned}
:\psi^{(\Lambda)^\dagger}(u)\psi^{(\Lambda)}(u+a): &= e^{ie(\Lambda(u+a)-\Lambda(u))} :\psi^\dagger(u)\psi(u+a): \\
&\quad -\frac{i}{2\pi a}\left(1 - e^{ie(\Lambda(u+a)-\Lambda(u))}\right).
\end{aligned} \tag{4.5}$$

As we saw in the simplest case, the inhomogeneous term arises because of the point splitting regularization without a Wilson line.

Similarly to the cases of the $U(1)$ current and energy-momentum tensor, we apply these transformation properties (4.3) and (4.5) to derive the expectation values of the higher-spin currents in the black hole background. From these equations, the expectation values of the generating function in the u coordinate with the gauge $A^{(u)}$ is related with that in the U coordinate with the gauge $A^{(U)}$ as follows,

$$\begin{aligned}
&\langle :\psi^\dagger(u)\psi(u+a):\rangle_{A^{(u)}} \\
&\quad = e^{-ie(\Lambda(u+a)-\Lambda(u))}\left[(\kappa U)e^{-\kappa a/2}\langle :\psi^{(U)^\dagger}(U(u))\psi^{(U)}(U(u+a)):\rangle_{A^{(U)}} + A_f(U,u)\right] \\
&\quad\quad -\frac{i}{2\pi a}\left(1 - e^{-ie(\Lambda(u+a)-\Lambda(u))}\right),
\end{aligned} \tag{4.6}$$

where the explicit form of $A_f(U,u)$ is

$$A_f(U,u) = \frac{i}{2\pi a}\left(\frac{\kappa a}{2}\frac{1}{\sinh\frac{\kappa a}{2}} - 1\right). \tag{4.7}$$

By imposing the regularity condition at the horizon $U = 0$, the fluxes at infinity are determined as

$$\langle : \psi^\dagger(u)\psi(u+a) : \rangle_{A^{(u)}} = \frac{i}{2\pi a}\left(e^{-i\frac{eQa}{r_+}}\frac{\kappa a}{2}\frac{1}{\sinh\frac{\kappa a}{2}} - 1\right). \tag{4.8}$$

This can be interpreted as the temperature-dependent part of a finite Green function for a charged fermion [26], as mentioned in Ref [22] in the case of Schwarzschild black holes. By expanding this as a power series of a, the fluxes of the higher-spin currents are obtained as

$$\begin{aligned}&\langle : i^n\psi^\dagger(u)\partial_u^n\psi(u) : \rangle_{A^{(u)}}\\ &\quad = \sum_{m=1}^{\lceil \frac{n}{2} \rceil}\left(\frac{eQ}{r_+}\right)^{n+1-2m}\frac{n!\,(1-2^{1-2m})\,B_m\kappa^{2m}}{2\pi(2m)!(n-2m+1)!} + \frac{1}{2\pi(n+1)}\left(\frac{eQ}{r_+}\right)^{n+1}.\end{aligned} \tag{4.9}$$

Here B_m is the Bernoulli number ($B_1 = 1/6,\ B_2 = 1/30$) and $\lceil x \rceil$ is the ceiling function, which returns the smallest integer not less than x.

It can be seen that these expectation values correspond to moments of the Fermi-Dirac distribution‡. Thermal radiations of a fermion with charge q from Reissner-Nordström black holes satisfy the Fermi-Dirac distribution $N_q(\omega)$ with a chemical potential corresponding to the value of the electric potential at the horizon,

$$N_q(\omega) = \frac{1}{e^{\beta(\omega - \frac{qQ}{r_+})} + 1}. \tag{4.10}$$

The flux of the higher-spin current (4.9) is equal to the following quantity

$$\int_0^\infty \frac{d\omega}{2\pi}\left[\omega^n N_e(w) - (-\omega)^n N_{-e}(w)\right]. \tag{4.11}$$

This can be interpreted as a moment of the thermal flux which consists of contributions from the fermion with charge e and its antiparticle with charge $-e$. Therefore one can reconstruct the whole information of the thermal radiation from the fluxes of the higher-spin currents derived in this section by using conformal field theory technique.

5 Conclusion and discussion

In this paper, we first generalized the method by Christensen and Fulling to charged black holes, and derived the fluxes of energy and charge by solving the conservation equations and anomaly equations. We then employed the conformal field theory method, used for

‡ Contributions from anti-holomorphic parts of the higher-spin currents to the fluxes vanish due to the boundary condition corresponding to the Unruh vacuum, i.e. no ingoing flux at infinity.

Schwarzschild black holes in Ref. [22], to charged black holes, and obtained these fluxes by investigating transformation properties of the gauge and energy-momentum currents under conformal and gauge transformations. It was crucial to consider the transformations from $(u, A^{(u)})$ to $(U, A^{(U)})$, where the former are suitable coordinate and gauge potential to evaluate quantities at infinity while the latter are suitable at the horizon. Finally we applied this method to higher-spin currents and calculated their expectation values in the charged black hole background. We showed that the expectation value of each higher-spin current in the Unruh vacuum exactly coincides with the corresponding specific moment of the thermal distribution.

The anomaly method has been universally applied to any type of black holes, black rings or cosmological models with horizons [4–20]. The essence is that the effective theories near horizons can be described by two-dimensional conformal fields in gravitational or gauge field backgrounds. The gauge field is either the original $U(1)$ gauge field or an effective gauge field induced by a dimensional reduction of higher-dimensional metric fields. Since our present analysis is also based on the same property near the horizon, the analysis of the higher-spin currents and derivations of the thermal radiations can be similarly applied to all kinds of black holes, black rings or cosmological models.

While we here used a conformal field theory technique, it would be possible to generalize the covariant calculation given in section 2 to the cases of higher-spin currents. To do this, it is necessary to construct covariant totally symmetric traceless higher-spin currents and to calculate the trace anomalies of these currents. Then the fluxes of these currents can be obtained by solving conservation equations and anomaly equations of the currents. In this analysis, relations between the covariant and conformal higher-spin currents, which are generalizations of eqs. (2.15) and (2.19), will be also obtained. We will discuss them in our future publication [25].

The derivation of Hawking radiations based on the anomaly method implies that the radiation is determined by the information of backgrounds at the horizon, not by the asymptotic charges measured at infinity. It would be interesting to see how a background which damps rapidly at infinity can affect the radiation.

Acknowledgements

We would like to thank H. Kodama and Y. Nakayama for useful information. We also thank A. Dhar, G. Mandal, S. R. Wadia and other members of the Tata Institute of Fundamental Research for their hospitality and stimulation during our stay.

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